WNE Linear Algebra Final Exam

Series A

10 March 2016

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and its series.**

Problem 1.
Let $V = \mathrm{lin}((1,1,0,-1,0),(0,1,1,0,2),(2,3,1,-2,2),(1,0,-1,-1,-2))$ be a subspace of $\mathbb{R}^5$.

a) find dimension of V and a system of linear equations which set of solutions is equal to V,

b) let $w = (1,4,3,-1,t) \in \mathbb{R}^5$. For which $t \in \mathbb{R}$ the subspace $\mathrm{lin}(w)$ is a subset of V, i.e. $\mathrm{lin}(w) \subset V$?

Problem 2.
Let $\alpha_1 = (1,0,2), \alpha_2 = (0,1,2), \alpha_3 = (1,0,3)$ be three vectors in $\mathbb{R}^3$.

a) which of the sequences below are ordered bases of $\mathbb{R}^3$? give a short explanation in each case

  i) $(\alpha_1, \alpha_2 - \alpha_1, 2\alpha_3)$,

  ii) (α_1, α_2),

  iii) $(\alpha_1, \alpha_1 + \alpha_2, \alpha_1 + \alpha_3, \alpha_2 + \alpha_3)$.

b) find coordinates of the vector $\alpha_1 + \alpha_2 - \alpha_3$ relative to the ordered basis $(\alpha_1 + \alpha_2, \alpha_2, \alpha_3)$.

Problem 3.
Let $A = \begin{bmatrix} s & 1 & 1 \\ 0 & 3 & 2 \\ 0 & -1 & 0 \end{bmatrix} \in M(3 \times 3; \mathbb{R})$.

a) check if matrix A is diagonalizable for $s = 2$,

b) for $s = 3$ find matrix $C \in M(3 \times 3; \mathbb{R})$ such that $C^{-1}AC = \begin{bmatrix} 3 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}$ for some $a, b \in \mathbb{R}$.

Problem 4.
Let $\mathcal{A} = ((1,2,0),(0,1,1),(1,2,1))$ be an ordered basis of $\mathbb{R}^3$ and let $\mathcal{B} = ((1,1),(1,0))$ and $\mathcal{C}$ be ordered bases of $\mathbb{R}^2$. The linear transformation $\varphi\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is given by the matrix $M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$.

The basis $\mathcal{C}$ of $\mathbb{R}^2$ is given by the matrix $M(id)_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$.

a) compute basis $\mathcal{C}$,
b) find formula of φ.

Problem 5.
Let $V = \operatorname{lin}((1,1,0),(0,1,1),(2,3,1))$ be a subspace of $\mathbb{R}^3$.
a) find an orthonormal basis of $V^\perp$,
b) compute the orthogonal projection of $w = (0,1,0)$ onto V.

Problem 6.
Let
$$A = \begin{bmatrix} 1 & 1 & r \\ 0 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$
a) for which $r \in \mathbb{R}$ the matrix A is invertible?,
b) for which $r \in \mathbb{R}$ the entry in the first row and the first column of the matrix A^{-1} is equal to -2?

Problem 7.
The affine space $H \subset \mathbb{R}^3$ is given by the equation $x_1 + x_2 + x_3 = 1$.
a) find a parametrization of the line L perpendicular to H and passing through $P = (2,0,1)$,
b) find the orthogonal projection of P onto H.

Problem 8.a) bring the following linear programming problem to a standard form
$x_1 - 3x_2 \longrightarrow max$
$$\begin{cases} x_1 - x_2 + 2x_3 \leqslant 1 \\ x_1 \geqslant 2 \\ x_2, x_3 \geqslant 0 \end{cases}$$
b) solve the following linear programming problem using simplex method
$2x_3 + 2x_4 \longrightarrow min$
$$\begin{cases} x_1 \qquad\qquad\quad + \ x_4 = 1 \\ \qquad x_2 \qquad\quad - \ x_4 = 2 \\ \qquad\qquad x_3 + 3x_4 = 6 \end{cases} \text{ and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 4$$